Problem 1.4

Use the cross product to find the components of the unit vector $\hat{\mathbf{n}}$ perpendicular to the shaded plane in Fig. 1.11.

Solution

Fig. 1.11 is shown below.

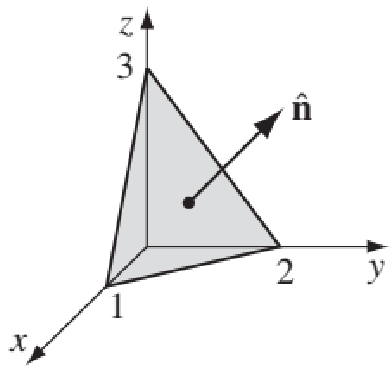
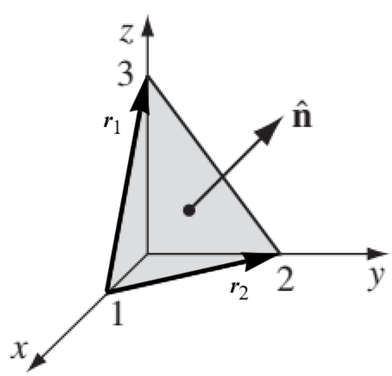


Fig. 1.11

Let $\mathbf{r}_1$ be the displacement vector from $(1, 0, 0)$ to $(0, 0, 3)$, and let $\mathbf{r}_2$ be the displacement vector from $(1, 0, 0)$ to $(0, 2, 0)$.



$$\mathbf{r}_1 = \langle 0, 0, 3\rangle - \langle 1, 0, 0\rangle = \langle -1, 0, 3\rangle$$
$$\mathbf{r}_2 = \langle 0, 2, 0\rangle - \langle 1, 0, 0\rangle = \langle -1, 2, 0\rangle$$

Take the cross product of $\mathbf{r}_2$ and $\mathbf{r}_1$ to get the vector perpendicular to the plane containing these vectors.

This perpendicular vector is $\mathbf{n}$; it points away from the origin by the right-hand corkscrew rule.

$$\begin{aligned}
\mathbf{n} &= \mathbf{r}_2 \times \mathbf{r}_1 \\
&= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} \\
&= \hat{\mathbf{x}} \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} - \hat{\mathbf{y}} \begin{vmatrix} -1 & 0 \\ -1 & 3 \end{vmatrix} + \hat{\mathbf{z}} \begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix} \\
&= \hat{\mathbf{x}}(6-0) - \hat{\mathbf{y}}(-3-0) + \hat{\mathbf{z}}(0+2) \\
&= 6\hat{\mathbf{x}} + 3\hat{\mathbf{y}} + 2\hat{\mathbf{z}}
\end{aligned}$$

Divide by the magnitude to get the corresponding unit vector.

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{6\hat{\mathbf{x}} + 3\hat{\mathbf{y}} + 2\hat{\mathbf{z}}}{\sqrt{6^2 + 3^2 + 2^2}} = \frac{6\hat{\mathbf{x}} + 3\hat{\mathbf{y}} + 2\hat{\mathbf{z}}}{7} = \frac{6}{7}\hat{\mathbf{x}} + \frac{3}{7}\hat{\mathbf{y}} + \frac{2}{7}\hat{\mathbf{z}}$$